CFL Phase of High Density QCD at Non-Zero Strange Quark Mass

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Abstract

We compute free energy of quark matter at asymptotically high baryon number density in the presence of non-zero strange quark mass including dynamics of pseudo Nambu-Goldstone bosons due to chiral symmetry breaking, extending previously existing analysis based on perturbative expansion in $m_s^2/4\mu\Delta$. We demonstrate that the CFLK^0 state has lower free energy than the symmetric CFL state for $0 < m_s^2/4\mu\Delta < 2/3$. We also calculate the spectrum of the fermionic quasiparticle excitations about the kaon condensed ground state in the regime $m_s^2/4\mu\Delta \sim 1$ and find that $(m_s^2/4\mu\Delta)_{crit} = 2/3$ for the CFL-gCFL phase transition, the leading order result reported in [1], is not modified.

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I. INTRODUCTION

It is known that while Color-Flavor Locked (CFL) phase is the ground state of 3-color, 3-flavor massless QCD at asymptotically large baryon number density [2, 3, 4], the ground state is modified by non-zero quark masses. An investigation of this issue has recently received a great deal of attention. See reviews [5, 6, 7, 8] and references therein.

In the CFL phase with zero quark masses, quarks of all three flavors participate in the BCS pairing due to the attractive one gluon exchange potential in the $\bar{3}$ channel. The resulting condensate spontaneously breaks the original $U(1)_B \times SU(3)_{Color} \times SU(3)_L \times SU(3)_R$ symmetry down to the diagonal subgroup, $SU(3)_{Color+L+R}$, causing the gauge bosons of the original $SU(3)_{Color}$ group to become massive by the Higgs mechanism [34]. A linear combination of gluon and the photon remains massless; this unbroken gauge symmetry is referred to as $U(1)_{\tilde{Q}}$, a "modified" electromagnetism. The low energy bosonic excitations are parametrized by $\Sigma = e^{2i\pi/f_\pi + i\eta'/\sqrt{6}f_{\eta'}}$ and $B = e^{i\beta/f_B}$ (with $\pi = \pi^a T^a$) where π_a's are the pseudo-scalar octet of Nambu-Goldstone bosons (NGB's) which arise from the breaking of chiral symmetry and T_a's are the fundamental representation $SU(3)$ generators; η' and β are NGB's due to breaking of the $U(1)_A$ (assumed to be a valid symmetry of the theory at high density) and $U(1)_B$ symmetries, respectively. The CFL symmetry breaking pattern is identical to that of QCD at low density [9] with the important difference between the two cases being that at high density the system is weakly coupled and the coefficients of the low energy EFT may be calculated from the underlying microscopic theory.

In the presence of non-zero quark masses the ground state changes in the following way. A relatively large strange quark mass encourages the system to reduce the strange quark number density relative to the density of up- and down-type quarks. The system responds to this stress by forming a condensate of collective excitations with quantum numbers of kaons in the ground state [10, 11, 12]. Kaon condensation allows the strange quark number density to be decreased without the costly breaking of Cooper pairs in the CFL background [35]. The $m_s^2/2\mu$ term, with μ being the quark number chemical potential, acts as a dynamically generated chemical potential for strangeness; when it exceeds the kaon mass the Bose-Einstein condensation of K^0 is triggered and the CFL ground state undergoes a second order phase transition to the CFLK^0 phase where the parity and hypercharge are spontaneously broken by the condensate. Recently it was argued that taking into account $\mathcal{O}(\alpha_s m_s^2)$ terms

in the effective lagrangian leads to more complicated phase diagram [13]. However, in this work we neglect this effect for the sake of simplicity.

So far the analysis has been performed to the leading order in $M^2/\mu\Delta$, where $M = \mathrm{diag}(m_u, m_d, m_s)$ is the quark mass matrix and Δ is the CFL gap parameter. It was shown that in the case of maximal kaon condensate (which occurs if we set $m_u = m_d = 0$ and $m_s \neq 0$) the free energy was lowered by $\delta\Omega = -f_\pi^2 \left(m_s^2/2\,\mu\right)^2/2 \propto m_s^4$ while the hypercharge density in the CFLK^0 ground state was $n_Y = f_\pi^2 \left(m_s^2/2\,\mu\right)$, where $f_\pi^2 = (21 - 8\,\mathrm{Log}\,2)\mu^2/36\,\pi^2$ is the leading order result for the pion decay constant [11, 12, 14]. Extrapolating into the regime $m_s^2/4\,\mu\,\Delta \sim 1$ one may expect that the kaon condensation free energy is of the same order as the BCS condensation energy itself and the number of strange quarks involved in the condensate is of the same order as $3\,\mu^2\Delta/\pi^2$, the (approximate) number of strange quarks involved in the BCS pairing. Therefore, it may be important to take into account dynamics of NGB's when one studies the properties of the ground state at large m_s.

Recently several calculations of dense quark matter free energy as a function of m_s have been performed and CFL-gapless CFL (gCFL) phase transition was observed at $(m_s^2/4\,\mu\,\Delta)_{crit} = 1/2$ [15, 16, 17]. In the gCFL phase the quark-quark pairing still involves all colors and flavors, but the spectrum contains massless fermionic modes. Similar phases were discussed in the context of two flavor QCD [18]. The objective of this work is to obtain better understanding of the CFL ground state as a function of m_s which should help better understanding of the CFL-gCFL phase transition and of the properties of gCFL phase.

In Section 2 of this article we calculate free energy of the CFLK^0 phase to all orders in the $m_s^2/4\,\mu\,\Delta$ expansion and show that the CFLK^0 persists as the energetically favored state even for $m_s^2/4\,\mu\,\Delta \sim 1$. In Section 3 we extend the leading order calculation of the CFL phase quasiparticle excitation spectrum about the CFLK^0 ground state and observe that the result reported in [1] is not modified by the higher order corrections.

II. FREE ENERGY CALCULATION

Let us first list some simplifying assumptions and approximations we make in this work

- We neglect small light quark masses and assume $M = \mathrm{diag}(0, 0, m_s)$. We work in the regime $m_s^2/\mu^2 \ll 1$, $m_s^2/4\Delta\mu \leq 1$ so that the light-strange pairing is still possible

[19]. This means that we neglect deviation of $v_s = p_s/\mu = \sqrt{1 - m_s^2/\mu^2}$, the strange quark Fermi velocity, from 1. Note that in this regime the chiral expansion, that is an expansion in $\partial/\Delta \sim m_s^2/\Delta\mu$, where one retains only few leading terms to attain desired accuracy, is expected to break down. In this regime the relevant effective theory should include both bosonic excitations of diquark condensate and quasiparticles as the degrees of freedom. We will use mean field approximation in which we will freeze the "radial" oscillations of the condensate magnitude while allowing oscillations of the condensate around the CFL vacuum state corresponding to the NGB's due to the chiral symmetry breaking in the massless case.

- Being in the weak coupling regime we will work to the leading order in α_s. Also we will neglect meson mass terms generated by quark masses. Quark mass terms connect particles and antiparticles and the resulting meson mass terms are generally suppressed by powers of $\Delta/\mu \propto \mathrm{Exp}(-const/g_s)$ and/or α_s [13, 14, 20].

- We assume that m_s is small enough so that we are in the CFL phase; we consider only color-flavor antisymmetric CFL gap term.

For excitation energies below μ the relevant degrees of freedom are the nonet of quasiparticles and holes, ten pseudo NGB's due to the spontaneous symmetry breaking of global symmetries and soft gauge bosons. The corresponding effective theory is called the High Density Effective Theory (HDET) [11, 21, 22, 23]. Let's consider the leading terms of the HDET Lagrangian of the CFL phase of high density QCD in the mean field approximation.

$$
\begin{aligned}
\mathcal{L} = {} & -2 \times \frac{3|\Delta|^2}{G} + \\
& + \mathrm{Tr}\left[L_v^\dagger(iv.\partial - (m_s^2/3\mu) - \mu_s^L - e\,v.A_{em}\,Q)L_v\right] - g_s\,\mathrm{Tr}\left(L_v^\dagger L_v v.A_c^T\right) + \\
& + \frac{\Delta}{2}\,\epsilon_{ijk}\epsilon_{abc}\,X^{kc}\,L_{-v\,\alpha}^{ia}L_{v\,\beta}^{jb}\,\epsilon^{\alpha\beta} + h.c. + (L \leftrightarrow R, X \leftrightarrow Y, \mu_s^L \leftrightarrow \mu_s^R) - \\
& - \frac{1}{2}\mathrm{Tr}\,G^{\mu\nu}G_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}.
\end{aligned}
\tag{1}
$$

Let us define all the quantities in (1)

- The first term is the mean field potential with coupling constant G being the coupling of the effective four fermion interaction generated by the hard gluon exchange.

- Fermionic fields $L_v^{ia}(x)$ and $R_v^{ia}(x)$ are the left- and right handed quasiparticle fields that annihilate states with Fermi velocity $v^\mu = (1, \hat{v})$, $\hat{v}.\hat{v} = 1$; flavor indices i, j, k

take values u, d, s; color indices a, b, c take values r, g, b; spinor indices $\alpha, \beta = \{1, 2\}$ are explicitly shown only in the gap term. Under the original symmetry group $SU(3)_c \times SU(3)_L \times SU(3)_R$, L_v transforms as $(3, 3, 1)$ and R_v transforms as $(3, 1, 3)$.

- We separated the SU(3) singlet part of the leading HDET mass term $M^2/2\mu$. The singlet term will only be responsible for the baryon density shift due to non-zero quark masses, but will not affect the m_s dependence of the condensation energy and from now on will be neglected. The remaining mass dependent terms are

$$
\begin{aligned}
\mu_s^L &= \frac{M M^\dagger}{2\mu} - \frac{1}{3} \mathrm{Tr} \frac{M M^\dagger}{2\mu}, \\
\mu_s^R &= \frac{M^\dagger M}{2\mu} - \frac{1}{3} \mathrm{Tr} \frac{M^\dagger M}{2\mu}.
\end{aligned}
\tag{2}
$$

With the M we use $\mu_s^L = \mu_s^R = -\mu_s T_8$, where $\mu_s = m_s^2/\sqrt{3}\mu$.

- The NGB fields are collected in $X^{kc} \sim \epsilon^{kij} \epsilon^{cab} \langle L_{ai}^* L_{bj}^* \rangle$ and $Y^{kc} \sim \epsilon^{kij} \epsilon^{cab} \langle R_{ai}^* R_{bj}^* \rangle$, 3×3 unitary matrix valued composite fields describing oscillations of the left and right handed quark-quark condensates about the CFL ground state. Under the original symmetry group $SU(3)_c \times SU(3)_L \times SU(3)_R$, X transforms as $(3, 3, 1)$ and Y transforms as $(3, 1, 3)$ [14, 24].

- $G^{\mu\nu}$ and $F^{\mu\nu}$ are the usual field strength tensors for the gauge fields A_c^μ and A_{em}^μ.

So, in our approximation the strange quark mass only appears in the role of an effective chemical potential for strangeness. An infinitesimal m_s induces kaon condensation in the ground state. What changes as m_s increases? As was stated above, the kaon mass terms generated by the quark masses are always suppressed by the powers of Δ/μ and/or α_s and cannot counterbalance the "chemical potential", $m_s^2/2\mu$, driving the K^0 Bose-Einstein condensation. For as long as the CFL pairing pattern is preferred, there is nothing that can significantly alter the K^0 condensate in the ground state as m_s increases. And in our approximation there is literally nothing that can change the form of the kaon condensate in the ground state. This is what our calculation will demonstrate.

Let us start by changing basis of quasiparticle fields to diagonalize the gap term. We write

$$
L_v = X \psi_L = \frac{1}{\sqrt{2}} \sum_{A=1}^{9} X \lambda^A L_v^A,
$$

5

$$R_v = Y \psi_R = \frac{1}{\sqrt{2}} \sum_{A=1}^{9} Y \lambda^A R_v^A, \tag{3}$$

where for $A = 1, \ldots, 8$ λ_A^{ia} are the Gell Mann matrices and $\lambda_9^{ia} = \sqrt{2/3}\,\delta^{ia}$ [11]. The gap term becomes

$$\mathcal{L}_{gap} = \frac{1}{2}\mathrm{Det}[X]\Delta_{AB}L_{-v\,\alpha}^A L_{v\,\beta}^B \epsilon^{\alpha\beta} + h.c. - (L \leftrightarrow R, \mathrm{X} \leftrightarrow \mathrm{Y}) \tag{4}$$

where $\Delta_{AB} = \frac{1}{2}\Delta(\lambda_A^{aa}\lambda_B^{bb} - \lambda_A^{ai}\lambda_B^{ia}) = -\Delta, A, B = 1..8$ and for $A, B = 9$, $\Delta_{AB} = \frac{1}{2}\Delta(\frac{2}{3} \times 3 \times 3 - \frac{2}{3} \times 3) = 2\Delta$ and we used $\epsilon_{abc}X^{am}X^{bn}X^{ck} = \mathrm{Det}[X]\epsilon^{mnk}$. In this calculation we neglect $U_B(1)$ and $U_A(1)$ NGB's and set $\mathrm{Det}[X] = \mathrm{Det}[Y] = 1$. The Lagrangian (1) may be rewritten as

$$\mathcal{L} = -2 \times \frac{3|\Delta|^2}{G} +$$
$$\sum_{A,B=1}^{9} \Psi_L^{A\dagger} \begin{pmatrix} i\,v.\partial\,\delta^{AB} + \mathcal{X}_{\hat{v}}^{AB} & \Delta_A\delta^{AB} \\ \Delta_A\delta^{AB} & i\,\tilde{v}.\partial\,\delta^{AB} - \mathcal{X}_{-\hat{v}}^{BA} \end{pmatrix} \Psi_L^B +$$
$$\sum_{A,B=1}^{9} \Psi_R^{A\dagger} \begin{pmatrix} i\,v.\partial\,\delta^{AB} + \mathcal{Y}_{\hat{v}}^{AB} & -\Delta_A\delta^{AB} \\ -\Delta_A\delta^{AB} & i\,\tilde{v}.\partial\,\delta^{AB} - \mathcal{Y}_{-\hat{v}}^{BA} \end{pmatrix} \Psi_R^B -$$
$$- \frac{1}{2}\mathrm{Tr}\,G^{\mu\nu}G_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{5}$$

where $\tilde{v} = (1, -\hat{v})$ and $p = \hat{v}.\vec{p} - \mu$. The rest of the definitions is as follows:

$$\Psi_L^A = \begin{pmatrix} L_v^A \\ (L_{-v}^A)^\dagger \end{pmatrix} \tag{6}$$

and

$$\Psi_R^A = \begin{pmatrix} R_v^A \\ (R_{-v}^A)^\dagger \end{pmatrix} \tag{7}$$

are the components of the Nambu-Gor'kov field;

$$\mathcal{X}_v^{AB} = \frac{1}{2}\mathrm{Tr}\,\lambda_A X^\dagger(iv.\partial - \mu_s^L - e\,v.A_{em}\,Q)X\lambda_B - \frac{1}{2}g_s\,\mathrm{Tr}\,\lambda_A\lambda_B\,v.A_c^T, \tag{8}$$

$$\mathcal{Y}_v^{AB} = \frac{1}{2}\mathrm{Tr}\,\lambda_A Y^\dagger(iv.\partial - \mu_s^R - e\,v.A_{em}\,Q)Y\lambda_B - \frac{1}{2}g_s\,\mathrm{Tr}\,\lambda_A\lambda_B\,v.A_c^T. \tag{9}$$

Before we proceed let us state the additional simplifying assumptions we make.

6

- We will neglect oscillations of the NGB fields about their ground state expectation values as the corresponding contribution to the free energy density is suppressed by $(\Delta/\mu)^2$ relative to the fermionic contribution. The values of X and Y in the ground state are to be determined by solving the corresponding equations of motion.

- We will also neglect oscillations of the gauge fields about their ground state expectation values. The $\langle A \rangle$'s are to be determined by solving the corresponding equations of motion which in the case of homogeneous quark matter are also the conditions for the gauge charge neutrality. These oscillations are important for the calculation of the gap parameter as the gap equation is dominated by the soft magnetic gluons [25], but here we do not attempt the calculation of the gap and neglect the corresponding terms in the effective action as being of higher order in the coupling expansion. Our approximation is then equivalent to working in the model with a color-flavor antisymmetric short range interactions in the mean field approximation (NJL model) [2, 3]. In our approximation the only role of the gauge fields is to ensure the gauge charge neutrality of the ground state [26, 27]. The spatial components of gauge fields do not have sources and, therefore, vanish in the ground state. Then the kinetic terms for the gauge fields vanish, too.

So, in this approximation we consider dynamics of quasiparticles in some gauge charge neutral meson field background. To calculate free energy we need to determine what this background is.

Starting from (5) let us formally integrate out fermions and then expand in powers of $\mathcal{X}$ and $\mathcal{Y}$. This is the usual EFT expansion; effectively we expand in $m_s^2/4\mu\Delta \lesssim 1$. Let us stress that we do not truncate this series at any finite order, that is, we do not rely on the smallness of the expansion parameter. We do rely on the analyticity of the free energy as a function of m_s; our analysis is valid within the radius of convergence of the expansion and breaks down at the point of a phase transition. We get the following effective action

$$
\begin{aligned}
\mathcal{S}[X, Y, A] \;=\; & -\int_x 2\frac{3|\Delta|^2}{G} - i\,\mathrm{Tr}\,Log\,\mathrm{S}^{-1} + \\
& + \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{2n}\mathrm{Tr}\left[S\begin{pmatrix} \tilde{\mathcal{X}}_v^{AB} & 0 \\ 0 & -\tilde{\mathcal{X}}_{-v}^{BA} \end{pmatrix} S\begin{pmatrix} \tilde{\mathcal{X}}_v^{AB} & 0 \\ 0 & -\tilde{\mathcal{X}}_{-v}^{BA} \end{pmatrix}\right]^{2n} +
\end{aligned}
$$

$$+ \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{2n} \text{Tr} \left[S \begin{pmatrix} \tilde{\mathcal{Y}}_v^{AB} & 0 \\ 0 & -\tilde{\mathcal{Y}}_{-v}^{BA} \end{pmatrix} S \begin{pmatrix} \tilde{\mathcal{Y}}_v^{AB} & 0 \\ 0 & -\tilde{\mathcal{Y}}_{-v}^{BA} \end{pmatrix} \right]^{2n}, \tag{10}$$

where

$$\tilde{\mathcal{X}}_v^{AB} = \frac{1}{2} \text{Tr} \, \lambda_A \left[X^\dagger (iv.\partial - \mu_s^L - v.A_{em} \, Q) X - v.A_c^T \right] \lambda_B \tag{11}$$

and

$$\tilde{\mathcal{Y}}_v^{AB} = \frac{1}{2} \text{Tr} \, \lambda_A \left[Y^\dagger (iv.\partial - \mu_s^R - v.A_{em} \, Q) Y - v.A_c^T \right] \lambda_B \tag{12}$$

and where S^{-1} is the inverse massless CFL propagator

$$S^{-1} = \begin{pmatrix} i\, v.\partial\, \delta^{AB} & \Delta_A \delta^{AB} \\ \Delta_A \delta^{AB} & i\, \tilde{v}.\partial\, \delta^{AB} \end{pmatrix}. \tag{13}$$

Also we have rescaled gauge fields to eliminate explicit dependence on the couplings. The operation Tr in (10) means trace over the 18 dimensional color flavor Nambu-Gor'kov space as well as over the space-time indices. The leading order terms in the expansion (10) produce leading order terms of the low energy EFT Lagrangian (for simplicity here we consider spatially uniform fields) [14, 24, 28]

$$\mathcal{L} = \frac{f_\pi^2}{2} \left[\text{Tr}(X^\dagger i D_0 X)^2 + \text{Tr}(Y^\dagger i D_0 Y)^2 \right] + \mathcal{O} \left(\mu^2 \Delta^2 \left(\frac{m_s^2}{\mu\,\Delta} \right)^4 \right), \tag{14}$$

where the covariant derivatives are given by

$$\begin{aligned} D_0 X &= \partial_0 X + i X A_0^{cT} + i A_0^{em} Q X + i \mu_s^L X, \\ D_0 Y &= \partial_0 Y + i Y A_0^{cT} + i A_0^{em} Q Y + i \mu_s^R Y. \end{aligned} \tag{15}$$

For spatially homogeneous time independent configurations neutral under $U_{\tilde{Q}}(1)$, $i.e.$ for $[X, Q] = 0$ and $[Y, Q] = 0$, it's easy to see that

$$\hat{A} = A_0^{cT} + A_0^{em} \, Q = -\frac{1}{2} (X^\dagger \mu_s^L X + Y^\dagger \mu_s^R Y), \tag{16}$$

the leading order solution, satisfies the equations of motion $\delta \mathcal{S}/\delta \, A_0 = 0$ for any term in the expansion (10) [27]. Since the CFLK^0 is devoid of electrons, we set $A_0^{em} = 0$. If we constrain the ground state to be the symmetric CFL configuration and set $X = 1$, $Y = 1$, then

$$\hat{A} = A_0^{cT} + A_0^{em} \, Q = -\mu_s \, T_8, \tag{17}$$

and $A_{em} = 0$, $A_c^3 = 0$, $A_c^8 = -\mu_s$.

The fact that the leading order solution $\hat{A}$ satisfies equations of motion at any order in the m_s expansion follows from the general form of gauge invariant effective action of a superconducting system [29].

Let us integrate out color gauge fields which in this approximation is achieved by simply substituting (16) back into the effective action. Now we have

$$\mathcal{S}[\Sigma] = -2 \int_x \frac{3|\Delta|^2}{G} - \frac{i}{2} \mathrm{Tr} \, \mathrm{L}og \left[\mathrm{S}^{-1} + J_V + J_A \right] \left[\mathrm{S}^{-1} + J_V - J_A \right], \qquad (18)$$

where

$$J_V = \frac{1}{4} \begin{pmatrix} \mathrm{Tr}\, \lambda_A [(X^\dagger \mu_s^L X + Y^\dagger \mu_s^R Y), \lambda_B] & 0 \\ 0 & -\mathrm{Tr}\, \lambda_B [(X^\dagger \mu_s^L X + Y^\dagger \mu_s^R Y), \lambda_A] \end{pmatrix} \qquad (19)$$

$$J_A = \frac{1}{4} \begin{pmatrix} \mathrm{Tr}\, \lambda_A (X^\dagger \mu_s^L X - Y^\dagger \mu_s^L Y) \lambda_B & 0 \\ 0 & -\mathrm{Tr}\, \lambda_B (X^\dagger \mu_s^L X - Y^\dagger \mu_s^L Y) \lambda_A \end{pmatrix} \qquad (20)$$

Considering only spatially homogeneous time independent configurations, we expand (18) in $m_s^2/4\mu\Delta$ and obtain

$$S[\Sigma] = -2 \int_x \frac{3|\Delta|^2}{G} - i \, \mathrm{Tr} \, \mathrm{L}og \, \mathrm{S}^{-1} + i \sum_{n=1}^{\infty} \frac{1}{2n} \mathrm{Tr} \, [\mathrm{S}\, J_A \, \mathrm{S}\, J_A]^{2n} . \qquad (21)$$

Here Tr also means trace over the 18 dimensional color flavor Nambu-Gor'kov space as well as over the space-time indices. The leading term in the expansion corresponds to the leading term of the CFL χPT Lagrangian

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{4} \mathrm{Tr} \, D_0 \Sigma^\dagger D_0 \Sigma + \mathcal{O} \left(\mu^2 \Delta^2 \left(\frac{m_s^2}{\mu \Delta} \right)^4 \right), \qquad (22)$$

where $\Sigma = XY^\dagger$ and $D_0 \Sigma = i\mu_s^L \Sigma - i\Sigma \mu_s^R$, the Bedaque-Schaefer covariant derivative for a constant field [11, 12]. We observe that J_V terms are not present in the expansion. One may show that

$$\Sigma = \Sigma_{K^0} = \xi_{K^0}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{pmatrix}, \qquad (23)$$

the leading order solution for the meson field [11, 12], satisfies equations of motion for Σ for any term in (21). From Σ we may determine values of matrices X and Y in the ground
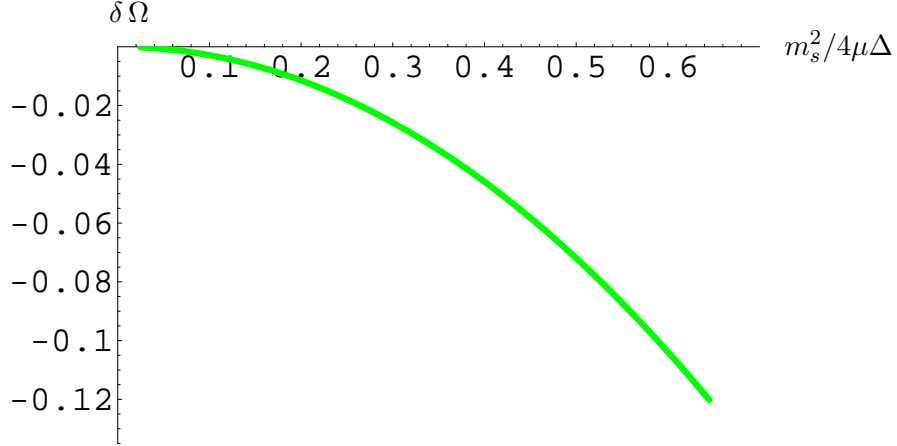
FIG. 1: Free energy difference $\delta\,\Omega = (\Omega(\Sigma = \Sigma_{K^0}) - \Omega(\Sigma = 1))/|\Omega^0_{CFL}|$, between $\Sigma = \Sigma_{K^0}$ and $\Sigma = 1$ ground states as a function of $m_s^2/4\mu\Delta$. The massless CFL BCS condensation energy, Ω^0_{CFL}, is defined in (26). The K^0 condensed state has lower value of free energy.

state which in the unitary gauge are given by $X_{K^0} = \xi_{K^0}$, $Y_{K^0} = \xi^\dagger_{K^0}$ [24, 27].

Now that we know the background gauge field values that ensure gauge charge neutrality and fields X and Y in the ground state, we may calculate the difference in free energies between CFLK^0 and the $SU(3)$ symmetric CFL ground state. The expression is

$$
\begin{aligned}
\delta\,\Omega \;&=\; \Omega(A = \hat{A}, \Sigma = \Sigma_{K^0}) - \Omega(A = \hat{A}, \Sigma = 1) = \\
&= \left[2 \times \frac{3|\Delta|^2}{G} + 2\pi \int_p \sum_{A=1}^{9} \left(\lambda_A^0(p; m_s) - \lambda_A(p; m_s, A = \hat{A}, \Sigma = \Sigma_{K^0}) \right) \right] - \\
&\quad - \left[2 \times \frac{3|\Delta|^2}{G} + 2\pi \int_p \sum_{A=1}^{9} \left(\lambda_A^0(p; m_s) - \lambda_A(p; m_s, A = \hat{A}, \Sigma = 1) \right) \right],
\end{aligned}
\tag{24}
$$

where $\{p0 - \lambda_A(p; m_s, A, \Sigma), p0 + \lambda_A(p; m_s, A, \Sigma)\}$ are the eigenvalues of the full inverse propagator in (5) and $\{p0 - \lambda_A^0(p; m_s), p0 + \lambda_A^0(p; m_s)\}$ are the eigenvalues for the charge neutral normal phase. Note that we don't need an explicit expression for the latter. We neglect the difference in the value of the gap parameter for the two cases which we have checked to be small. Alternatively, we calculated the difference in free energy starting from (18). The results are shown in Fig. 1. The K^0 condensed configuration has lower free energy relative to the symmetric CFL state even for $m_s^2/4\mu\Delta \sim 1$. The result of hypercharge density calculation in the CFLK^0 phase is shown in Fig. 2. We note that the numerical results are surprisingly close to the leading order EFT predictions.
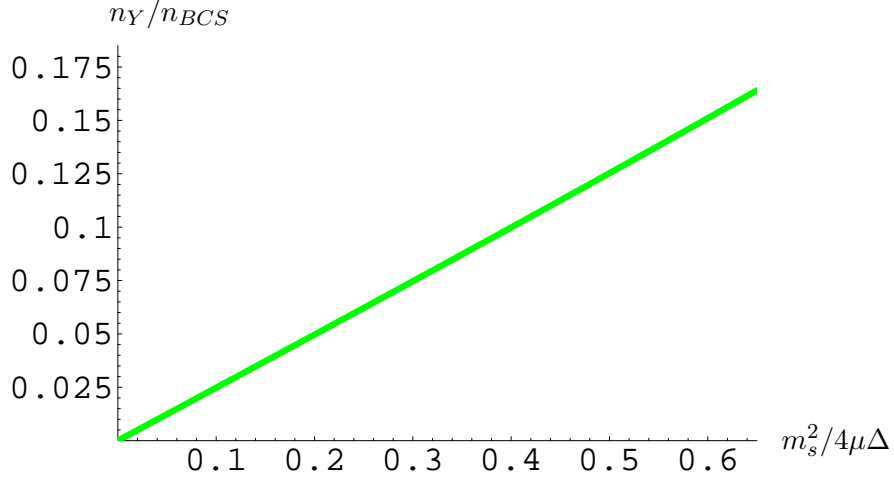
FIG. 2: The hypercharge density n_Y/n_{BCS} in the CFLK^0 ground state as a function of $m_s^2/4\mu\Delta$, where $n_{BCS} = 3\,\mu^2\,\Delta/\pi^2$, the approximate concentration of quarks of a given flavor involved in the BCS pairing. The result is close to the leading order EFT prediction.

In this calculation we performed the free energy renormalization by trading the bare coupling $1/G$ for the (physical) superconducting gap parameter Δ_0 of the massless CFL by expressing $1/G$ from the gap equation [30]

$$\left.\frac{\delta\Omega}{\delta\Delta}\right|_{m_s=0,\Delta=\Delta_0} = 0. \tag{25}$$

Then, for example, the massless CFL BCS condensation free energy is given by [3]

$$\Omega^0_{CFL} = -3\,\mu^2\,\Delta_0^2/\pi^2. \tag{26}$$

III.   QUASIPARTICLE SPECTRUM

Based on the results obtained, in this Section we extend the leading order calculation of the quasiparticle spectrum in the presence of kaon condensed background performed in [1] into the regime $m_s^2/4\mu\Delta \sim 1$. Starting from (18) we write down the leading terms of the effective Lagrangian for the nonet of quasiparticle excitations about CFLK^0 ground state

$$\begin{aligned}
\mathcal{L} = {} & \mathrm{Tr}\,\psi^\dagger_{Lv}iv.D\,\psi_{Lv} + \mathrm{Tr}\,\psi^\dagger_{Lv}\mathcal{A}_0\,\psi_{Lv} + \frac{\Delta_{AB}}{2}\psi^A_{L-v}\,\psi^B_{Lv} + h.c. + \\
& \mathrm{Tr}\,\psi^\dagger_{Rv}iv.D\,\psi_{Rv} - \mathrm{Tr}\,\psi^\dagger_{Rv}\mathcal{A}_0\,\psi_{Rv} - \frac{\Delta_{AB}}{2}\psi^A_{R-v}\,\psi^B_{Rv} + h.c.,
\end{aligned} \tag{27}$$

11

where the nonet field ψ has been defined in (3). This Lagrangian may be rewritten in the form prescribed by the chiral perturbation theory for the baryon fields [31]

$$
\begin{aligned}
\mathcal{L} &= \mathrm{Tr}\,\psi_v^\dagger iv.D\psi_v + \bar{D}\,\mathrm{Tr}\,\psi_v^\dagger \gamma_5 \left\{ v.\mathcal{A}, \psi_v \right\} + \bar{F}\,\mathrm{Tr}\,\psi_v^\dagger \gamma_5 \left[v.\mathcal{A}, \psi_v \right] + \\
&\quad + \frac{\Delta_{AB}}{2}\psi_{-v}^A\, C\,\gamma_5\,\psi_v^B + h.c.,
\end{aligned}
\tag{28}
$$

with $\bar{D} = \bar{F} = 1/2$ and C being the charge conjugation operator. The covariant derivative here is $D_\mu \psi = \partial_\mu \psi + i[\mathcal{V}_\mu, \psi]$; the time components of the vector and axial currents modified by the quark mass terms are

$$
\mathcal{V}_0 = \frac{i}{2}\left(\xi^\dagger (\partial_0 + i\,\mu_s^L)\,\xi + \xi\,(\partial_0 + i\,\mu_s^R)\,\xi^\dagger \right) = -\frac{1}{2}\left(\xi^\dagger \mu_s^L \xi + \xi \mu_s^R \xi^\dagger \right),
\tag{29}
$$

and

$$
\mathcal{A}_0 = \frac{i}{2}\left(\xi^\dagger (\partial_0 + i\,\mu_s^L)\,\xi - \xi\,(\partial_0 + i\,\mu_s^R)\,\xi^\dagger \right) = -\frac{1}{2}\left(\xi^\dagger \mu_s^L \xi - \xi \mu_s^R \xi^\dagger \right).
\tag{30}
$$

Shown in Fig. 3 are the masses of the quasiparticles that become gapless as a function of $m_s^2/4\mu\Delta$. The $\Sigma = 1$ case develops gapless modes at $m_s^2/4\mu\Delta = 1/2$, while a CFLK^0 mode becomes gapless at $m_s^2/4\mu\Delta = 2/3$. A detailed description is presented in [1]. Here we only observe that the leading order in $m_s^2/4\mu\Delta$ result and the result of the full calculation coincide. It follows from the fact that in the approximation we use the leading order in $m_s^2/4\mu\Delta$ expressions for the background gauge fields and Σ are not modified by the higher order terms. It has been argued that the appearance of a gapless mode signifies the point of CFL-gCFL phase transition [15]. Our calculation confirms that due to the nontrivial flavor alignment in the ground state the CFL-gCFL phase transition is expected at $(m_s^2/4\mu\Delta)_c = 2/3$ which is $4/3$ times higher than the symmetric CFL value.

Note that a non analyticity at m_s^{crit} is built into the expression for free energy (24) since at $m_s = m_s^{crit}$ the momentum integral over $\lambda_A > 0$ has to be modified to take into account the breach in the quasiparticle spectrum. Then the value of $m_s^{crit} = \sqrt{8\,\mu\,\Delta/3}$ is the convergence radius of the m_s series expansions (10), (21) which is consistent with the fact that at this point the system undergoes a phase transition.

We have neglected the quantum mechanical oscillations of the collective bosonic fields about CFLK^0 ground state. One may have to worry about the stability of the ground state since as m_s grows several quasiparticles are becoming lighter, whereas the massive mesonic excitations about CFLK^0 ground state have masses of order $m_s^2/2\,\mu$ and at some
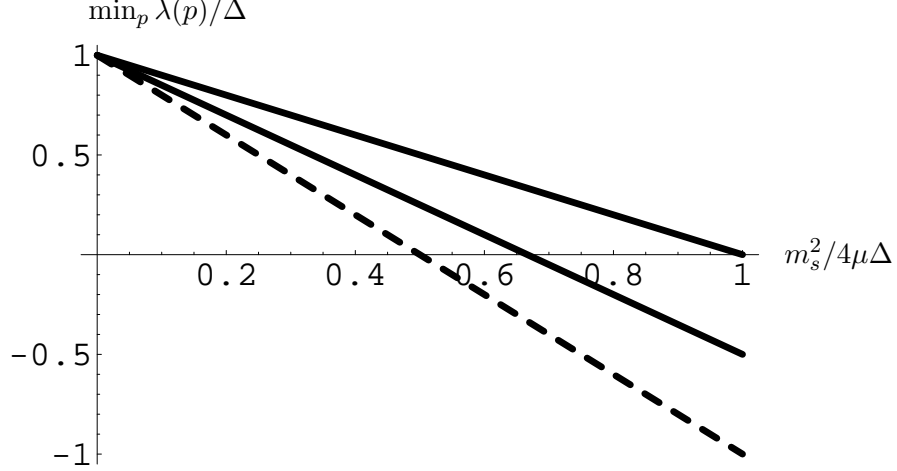
FIG. 3: Masses of the quasiparticles that are lowered due to the presence of m_s as a function of $m_s^2/4\mu\Delta$. The dashed line depicts a doubly degenerate $\Sigma = 1$ case mode, solid lines are the $\Sigma = \Sigma_{K^0}$ case modes. The results are in agreement with [1].

large enough m_s a heavy Goldstone may decay into a particle-hole pair. However, due to the hypercharge and isospin charge conservation the quasiparticle-hole pair in the fermionic loop may not go on-shell for any m_s and the ground state is stable.

IV. DISCUSSION: SOME OPEN QUESTIONS

We have demonstrated that taking into account dynamics of pseudo NGB's in the CFL phase of high density QCD and allowing a nontrivial flavor alignment in the ground state gives a sizable decrease in the free energy in the regime $m_s^2/\mu\Delta \sim 1$. It is remarkable that we were able to find a sensible approximation where it was possible to do the calculation to all orders in the m_s expansion. We also extended the leading order calculation of quasiparticle spectrum modified by the presence of kaon condensed background into the regime $m_s^2/4\mu\Delta \sim 1$ and found the leading order result unmodified.

Several comments may be made based on the results of this work. First, since the CFL-gCFL phase transition is continuous, the kaon condensate in the gCFL ground state cannot be turned off abruptly and will persist at least for some values of $m_s > m_s^{crit}$. While we argued that in the CFL nothing can significantly alter the magnitude of the kaon condensate, the non-zero non CFL gap terms, Δ_1 and Δ_3 defined in (31), may just do that by generating

mass terms for the kaons. The question whether K^0 condensate will persist in the gCFL phase requires further investigation.

The most interesting issue that we have not addressed in this work is the investigation of the CFL-gCFL phase transition and of the gCFL ground state including dynamics of pseudo NGB's. In such a calculation one should allow deviations from the CFL pairing pattern in the gap term. For example, one may consider

$$\Delta^{m\,k} = \Delta_{CFL}\delta^{m\,k} + \Delta_{13}^{m\,k}, \tag{31}$$

with

$$\Delta_{13}^{m\,k} = \mathrm{d}iag(\Delta_1, 0, \Delta_3). \tag{32}$$

This pairing ansatz introduced by Alford *et al* in [15] is the simplest one that allows both disruption of the light-strange pairing ($\Delta_3 \neq 0$) which is directly induced by m_s, and of the pairing between light quarks ($\Delta_1 \neq 0$) which is due to the gauge charge neutrality and which, of course, is also indirectly induced by the strange quark mass. Unlike the CFLK^0 case, in the presence of the non CFL gap terms one does not know anymore the exact values of the background gauge fields and Σ in the ground state. A perturbation theory in $\Delta_{1,3}$ about the CFLK^0 state would seem to be a reasonable approach. To perform this calculation one needs to construct a low energy EFT in the regime $m_s^2/4\mu\Delta \sim 1$ describing dynamics of light bosonic and fermionic modes.

In this simplified calculation we treated gluons as classical background fields. It would be interesting to repeat the calculation of the CFL free energy including the effects of quantum mechanical oscillations of the gluon fields into account.

These questions are left for future work. The full answer to the question: "What is the ground state of dense QCD as a function of quark masses?" is yet to come.

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(D.Y.).

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[35] In dense hadronic matter, kaon condensation involves K^- mesons, which introduce strange quarks into the system [33] allowing one to approach $SU(3)$ neutrality at high density. In contrast, the CFL$K0$ phase involves K^0 condensation which reduces the net number of strange quarks participating in the BCS pairing, driving the system away from $SU(3)$ neutrality in response to a large strange quark mass.